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Tau Polarization in $\Lambda_b \to X_c \tau \bar{\nu}$ and $B \to X_c \tau \bar{\nu}$

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Abstract

We discuss the longitudinal and transverse τ-polarization in inclusive decays of hadrons containing b-quarks. The calculation is performed by means of an OPE in HQET. Some mathematical difficulties in calculating transverse polarizations are explained. Numerical results are presented for longitudinal and for transverse polarizations, both in and perpendicular to the decay plane.

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I Introduction

In this paper we present a detailed discussion of lepton polarization in the inclusive decay of the B-meson and the Λ_b. Our calculations make use of the heavy quark effective theory (HQET) and an operator product expansion (OPE). These techniques were initially developed for massless leptons in the final state [1] [2] [3] [4]. Recently massive leptons in B decays have also been considered [5] [6] and their longitudinal polarization has been calculated in [7]. We will supplement this discussion by a calculation of the longitudinal τ-polarization in the decay $\Lambda_b \to X_c \tau \bar{\nu}$ and the transverse τ-polarization in B and Λ_b decays.

In the calculation of transverse polarizations one encounters certain divergent integrals if the decay width is analysed as a function of the momentum transfer q^2. In [8] the same divergences have been encountered in the calculation of a CP violating matrix element for transverse τ- polarization. In that work the authors introduce a regularization procedure in order to obtain finite results. We will show that these divergences can be bypassed by using an alternative parametrization that yields mathematically well defined quantities. A similar mathematical problem is responsible for the occurrence of δ-functions and their derivatives in the differential decay widths calculated in [3]. Our parametrization removes this problem as well. Thus all seemingly unphysical aspects of the inclusive rates, i.e. δ-functions and divergences in polarizations, are of mathematical origin and can be avoided.

The transverse polarization can be viewed as an additional test of HQET and the operator product expansion. For instance, a CP conserving matrix element for B-meson decays should not produce any polarization transverse to the decay plane. This is confirmed by our calculations. On the other hand, for polarized Λ_b decay, there is a nontrivial transverse τ-polarization which is correlated with the Λ_b-spin, and contains components both in and perpendicular to the decay plane. Experimentally polarizations can be measured and compared to theoretical predictions without any knowledge of the CKM matrix element $|V_{cb}|$. Also the fifth power of the somewhat hazy b-quark mass does not appear in the results, thereby reducing the uncertainties of the predictions further. On a more technical level the calculation of transverse polarization is an interesting test of a theory that claims to describe small corrections to the free quark model.

Mainly to establish notation, we briefly review some of the basic steps of this calculation. If not stated otherwise we will use the notations established in [3]. The decay widths are of the form

$$d\Gamma = 2|V_{cb}|^2 G_F^2 2\pi L_{\mu\nu} H^{\mu\nu} \frac{d^3 P_\nu}{(2\pi)^3 2P_\nu^0} \frac{d^3 P_\tau}{(2\pi)^3 2P_\tau^0} \qquad (1)$$

Subsequently we will use them normalized to the free quark model total decay width $\Gamma_0 = \frac{|V_{cb}|^2 G_F^2 m_b^5}{192\pi^3}$. The hadron tensor is given by the imaginary part of the matrix

element of a transition operator.

$$H_{\mu\nu} = -\frac{1}{\pi}\mathrm{Im}\, T_{\mu\nu} \tag{2}$$

Both sides of this equation can be decomposed into form factors multiplied by Lorentz structures. The most general decomposition excluding terms violating time reversal invariance is

$$\begin{aligned} T^{\mu\nu} &= -g^{\mu\nu}T_1 + v^\mu v^\nu T_2 - i\epsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta T_3 + q^\mu q^\nu T_4 + (q^\mu v^\nu + q^\nu v^\mu)T_5 \\ &\quad -(qs_h)\left(-g^{\mu\nu}S_1 + v^\mu v^\nu S_2 - i\epsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta S_3 + q^\mu q^\nu S_4 + (q^\mu v^\nu + q^\nu v^\mu)S_5\right) \\ &\quad +(s_h^\mu v^\nu + s_h^\nu v^\mu)S_6 + (s_h^\mu q^\nu + s_h^\nu q^\mu)S_7 + i\epsilon^{\mu\nu\alpha\beta}v_\alpha s_{h\beta}S_8 + i\epsilon^{\mu\nu\alpha\beta}q_\alpha s_{h\beta}S_9 \end{aligned} \tag{3}$$

q is the total momentum of the lepton pair, v is the 4-velocity of the decaying hadron and s_h is the hadron spin vector. All form factors are functions of q^2 and qv. For the hadrons under consideration here they can be calculated by means of an operator product expansion in the heavy quark effective theory. $L_{\mu\nu}$ is the usual lepton tensor

$$L^{\mu\nu} = 8\left(P_\tau^\mu P_\nu^\nu + P_\tau^\nu P_\nu^\mu - g^{\mu\nu}(P_\tau P_\nu) + i\epsilon^{\mu\nu\alpha\beta}P_{\tau\alpha}P_{\nu\beta}\right) \tag{4}$$

For polarized leptons it is convenient to introduce

$$L^s_{\mu\nu} = Tr[(\not{P}_\tau + m_\tau)\frac{1+\gamma^5\not{s}_\tau}{2}\gamma^\mu(1-\gamma^5)\not{P}_\nu\gamma^\nu(1-\gamma^5)] \tag{5}$$

$$\begin{aligned} L^p_{\mu\nu} &= \frac{1}{2}(L^s_{\mu\nu} - L^{-s}_{\mu\nu}) \\ &= -4m_\tau\left(P_\nu^\nu s_\tau^\mu + P_\nu^\mu s_\tau^\nu - g^{\mu\nu}(P_\nu s_\tau) - i\epsilon^{\mu\nu\alpha\beta}P_{\nu\alpha}s_{\tau\beta}\right) \end{aligned} \tag{6}$$

and to calculate the matrix element obtained by contracting the hadron tensor with (6). Then the τ-polarization in s_τ direction is given by

$$2\frac{d\Gamma^p}{d\Gamma} \tag{7}$$

where $d\Gamma^p$ is the decay width calculated with the lepton tensor (6), and $d\Gamma$ with (4).

The paper is organized as follows: In section II the OPE is discussed. Section III is devoted to the calculations of the longitudinal τ-polarization in the decay $\Lambda_b \to X_c\tau\bar{\nu}$. A detailed discussion of the transverse polarization in Λ_b decays, including a discussion of the mathematical difficulties, will be given in section IV. The same calculations are repeated for B-meson decays in section V. Section VI contains the numerical evaluation.

II The Operator Product Expansion

We recapitulate briefly the operator product expansion in the context of HQET [1] [3] [4] [6] [9]. A convenient way to perform the expansion makes use of the fact that

the heavy quark propagator can be written as a geometric series. The starting point for constructing an OPE is the transition operator

$$\hat{T}^{\mu\nu} = -i \int d^4x \mathrm{e}^{-iqx} T[j^{\mu\dagger}(x) j^\nu(0)] \tag{8}$$

which takes the form

$$\hat{T}^{\mu\nu} = \bar{b}\gamma^\mu P_L \frac{1}{\not{P}_b - \not{q} - m_c} \gamma^\nu P_L b \tag{9}$$

for $b \to c$ transitions. Here P_L is the left handed projection operator, P_b is the b-quark momentum and $q = P_\tau + P_\nu$ is the total momentum of the τ and the ν. To obtain the transition amplitude $T_{\mu\nu}$ in (2) this operator is placed between B-meson or Λ_b states. The OPE expresses the propagator in the transition operator through a sum of operators. To construct this expansion one interprets the b-quark momentum as an operator $P_b^\mu \to i\partial^\mu + g_s A^\mu$ which contains the gluon field A because the propagating c-quark cannot be viewed as free. It is exposed to the background field generated by interactions with the light degrees of freedom in the hadron. The full QCD b-field in (9) is related to the field in HQET through

$$b = \mathrm{e}^{-im_b vx}(1 + \frac{i\not{D}}{2m_b} + \cdots) b_v \tag{10}$$

The b-quark momentum has been decomposed into a large part due to the hadron's motion and a small part due to interactions between the heavy quark and the light quarks: $P_b = m_b v + k$. v is the 4-velocity of the meson and k is the momentum due to interactions between the quarks. Between b-quark fields the b-momentum operator $\hat{P}_b$ can be replaced: $\hat{P}_b \to m_b v + \hat{k}$. Now the gluon field is contained in $\hat{k} = i\partial + g_s A$. The operator $\hat{k}$ produces only the residual momentum of $\mathcal{O}(\Lambda_{QCD})$. It obeys the commutator relation: $[\hat{k}^\mu, \hat{k}^\nu] = ig_s G^{\mu\nu}$, which implies that the gluon tensor is of second order in k. The OPE can be performed if the propagator is rewritten as

$$\frac{1}{m_b\not{v} + \hat{\not{k}} - \not{q} - m_c} = (\not{V}_0 + \hat{\not{k}} + m_c)\frac{1}{(V_0 + \hat{k})^2 + \frac{1}{2}g_s\sigma^{\alpha\beta}G_{\alpha\beta} - m_c^2} \tag{11}$$

with $V_0 = m_b v - q$ and $\sigma^{\alpha\beta} = \frac{i}{2}(\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha)$. Defining $\Delta_0 = V_0^2 - m_c^2$ the fraction can be rewritten in a form suitable for expansion in a geometric series

$$\begin{aligned}
\frac{1}{(V_0 + \hat{k})^2 + \frac{1}{2}g_s\sigma^{\alpha\beta}G_{\alpha\beta} - m_c^2} &= \frac{1}{\Delta_0}\frac{1}{1 + \frac{2V_0\hat{k} + \hat{k}^2 + \frac{1}{2}g_s\sigma^{\alpha\beta}G_{\alpha\beta}}{\Delta_0}} \\
&= \frac{1}{\Delta_0}\sum_{n=0}^{\infty}\left(-\frac{2V_0\hat{k} + \hat{k}^2 + \frac{1}{2}\sigma^{\alpha\beta}G_{\alpha\beta}}{\Delta_0}\right)^n \qquad (12)
\end{aligned}$$

This expansion corresponds to an expansion in $\frac{\hat{k}}{m_b}$ since Δ_0 is of $\mathcal{O}(m_b^2)$. Inserting this expansion in (11) and using the algebra of the γ-matrices gives the expansion

of the propagator in powers of $\hat{k}$. Expressions containing the charm mass do not contribute because of the projection operators in the amplitude (9).

$$\begin{aligned}\frac{1}{\not{V}_0 + \hat{\not{k}} - m_c} &= \frac{\not{V}_0}{\Delta_0} + \frac{\hat{\not{k}}}{\Delta_0} - 2\frac{\not{V}_0 V_0 \hat{k}}{\Delta_0^2} - 2\frac{\gamma^\delta V_0^\tau \hat{k}_{(\delta}\hat{k}_{\tau)}}{\Delta_0^2} \\ &\quad - \frac{\not{V}_0 \hat{k}^2}{\Delta_0^2} + 4\frac{\not{V}_0 (V_0 \hat{k})^2}{\Delta_0^3} + \frac{g_s}{2\Delta_0^2} V_{0\alpha}\gamma_\beta\gamma_5\epsilon^{\alpha\beta\delta\tau}G_{\delta\tau} + \mathcal{O}(\hat{k}^3) \qquad (13)\end{aligned}$$

The brackets around indices indicate a symmetrized expression. These operators are identical to those found in [3]. The method of obtaining the matrix elements of these operators is explained in detail in that paper. We will not repeat that procedure here.

III Longitudinal τ-Polarization in Λ_b decays

The calculation of the longitudinal τ-polarization in decays of polarized Λ_b's can be done in precisely the same way as the inclusive rates [3] [4] and the longitudinal polarization for B-meson decays [7] were calculated. We will use that method here without describing the details of the procedure. For massive leptons a number of form factors in addition to those given in [3] are necessary to calculate the decay rates. In the notation of Manohar and Wise [3], which we use throughout this paper, they are (to $\mathcal{O}(\hat{k}^2)$)

$$T_4 = \frac{4}{3}m_b(K+G)\frac{1}{\Delta_0^2} \qquad (14)$$

$$\begin{aligned}T_5 &= -\frac{1}{2}\frac{1}{\Delta_0} - \frac{1}{3}m_b K(4m_b + 5qv)\frac{1}{\Delta_0^2} - \frac{5}{3}m_b G qv\frac{1}{\Delta_0^2} \\ &+ \quad \frac{4}{3}m_b^2 K(q^2 - (qv)^2)\frac{1}{\Delta_0^3} \qquad (15)\end{aligned}$$

$$S_4 = 0 \qquad (16)$$

$$S_5 = -\frac{2}{3}m_b K\frac{1}{\Delta_0^2} \qquad (17)$$

$$S_7 = \frac{1}{2}(1+\epsilon)\frac{1}{\Delta_0} + m_b K(qv + \frac{2}{3}m_b)\frac{1}{\Delta_0^2} - \frac{4}{3}m_b^2 K(q^2 - (qv)^2)\frac{1}{\Delta_0^3} \qquad (18)$$

ϵ is a spin symmetry breaking parameter. Its value can be estimated through sum rule calculations. Sum rules [9] [3] also give a value for the constant

$$K = -\langle H_b|\bar{b}_v\frac{(iD)^2}{2m_b^2}b_v|H_b\rangle = 0.01 \qquad (19)$$

We use this value for B-mesons and Λ_b even though it is by no means a universal constant. The matrix element of the chromomagnetic operator between B-meson

states is

$$G = \langle B|\bar{b}_v \frac{g_s G_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b^2} b_v|B\rangle = -0.0065 \tag{20}$$

but vanishes between Λ_b states. For the longitudinal polarization the spin projection vector s_τ in (6) is set to

$$s_\tau = \left(\frac{|\underline{P}_\tau|}{m_\tau}, \frac{P_\tau^0 \underline{P}_\tau}{m_\tau |\underline{P}_\tau|}\right) \tag{21}$$

The decay widths, normalized to the free quark total decay width and transformed to normalized variables $x = \frac{2P_\nu^0}{m_b}, y = \frac{2P_\tau^0}{m_b}, \hat{q}^2 = \frac{q^2}{m_b^2}$

$$\frac{1}{\Gamma_0} d\Gamma = \frac{3}{2}\frac{1}{m_b} L_{\mu\nu} H^{\mu\nu} dx dy d\hat{q}^2 d\cos(\Theta_s) \tag{22}$$

$$\frac{1}{\Gamma_0} d\Gamma^p = \frac{3}{2}\frac{1}{m_b} L_{\mu\nu}^p H^{\mu\nu} dx dy d\hat{q}^2 d\cos(\Theta_s) \tag{23}$$

can be calculated as shown in [3]. Θ_s is the angle between the spin vector of the Λ_b and the direction of the τ 3-momentum. After integration over x and $\hat{q}^2$ the spin-independent part of the decay width can be written symbolically as

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy d\cos(\Theta_s)} = 2\pi\left(A_0 + A_1 K + (B_0(1+\epsilon) + B_1 K)\cos(\Theta_s)\right) \tag{24}$$

The functions A_0, A_1, B_0 and B_1 are given by

$$\begin{aligned}
A_0 &= \frac{1}{2\pi}\sqrt{y^2 - 4\rho_\tau}\left((-3y^2 + 6y(1+\rho_\tau) - 12\rho_\tau)x_0^2\right.\\
&\quad \left. +(y^2 - 3y(1+\rho_\tau) + 8\rho_\tau)x_0^3\right) && (25)\\
A_1 &= \frac{1}{2\pi}\frac{\sqrt{y^2 - 4\rho_\tau}}{(1+\rho_\tau - y)^2}\left(-2(y-2)(y-2\rho_\tau)(y^2 - 4\rho_\tau)\right. && (26)\\
&\quad -4(y^4 - 3y^3(1+\rho_\tau) + y^2(5 + 6\rho_\tau + 5\rho_\tau^2) - 12(1+\rho_\tau)\rho_\tau y\\
&\quad -8\rho_\tau(1 - 4\rho_\tau + \rho_\tau^2))x_0\\
&\quad +2(2y^4 - 8(1+\rho_\tau)y^3 + (15 + 28\rho_\tau + 15\rho_\tau^2)y^2 - 52\rho_\tau(1+\rho_\tau)y\\
&\quad -18\rho_\tau(1 - 6\rho_\tau + \rho_\tau^2))x_0^2\\
&\quad -\frac{4}{3}(y^4 - 5y^3(1+\rho_\tau) + 2y^2(5 + 11\rho_\tau + 5\rho_\tau^2) - 40\rho_\tau(1+\rho_\tau)y\\
&\quad \left. -2\rho_\tau(5 - 38\rho_\tau + 5\rho_\tau^2))x_0^3\right)\\
B_0 &= \frac{1}{2\pi}\left(-3(y^3 - 2y^2\rho_\tau - 4y\rho_\tau + 8\rho_\tau^2)x_0^2\right. && (27)\\
&\quad \left. +(y^3 + y^2(1 - 3\rho_\tau) - 4y\rho_\tau - 4(1 - 3\rho_\tau)\rho_\tau)x_0^3\right)\\
B_1 &= \frac{1}{2\pi}\frac{y^2 - 4\rho_\tau}{(1+\rho_\tau - y)^2}\left(-2(y - 2\rho_\tau)(y^2 - 4\rho_\tau)\right. && (28)
\end{aligned}$$

$$
\begin{aligned}
&-4(y^3-(4+3\rho_\tau)y^2+(5\rho_\tau+11)\rho_\tau y+6\rho_\tau(1-3\rho_\tau))x_0\\
&+2(2y^3-2(3+4\rho_\tau)y^2+(-5+22\rho_\tau+15\rho_\tau^2)y+24\rho_\tau(1-2\rho_\tau))x_0^2\\
&-\frac{4}{3}(y^3-(2+5\rho_\tau)y^2+(-5+11\rho_\tau+10\rho_\tau^2)y+6\rho_\tau(3-5\rho_\tau))x_0^3\Big)
\end{aligned}
$$

The polarization-dependent width has a similar decomposition

$$
\frac{1}{\Gamma_0}\frac{d\Gamma^p}{dyd\cos(\Theta_s)}=2\pi\left(a_0+a_1K+(b_0(1+\epsilon)+b_1K)\cos(\Theta_s)\right) \tag{29}
$$

with

$$
\begin{aligned}
a_0 &= \frac{1}{2\pi}(y^2-4\rho_\tau)\left(\frac{3}{2}(y-2)x_0^2+\frac{1}{2}(3-\rho_\tau-y)x_0^3\right) && (30)\\
a_1 &= \frac{1}{2\pi}\frac{(y^2-4\rho_\tau)}{(1+\rho_\tau-y)^2}\Big((y^3-2y^2-4\rho_\tau y+8\rho_\tau) && (31)\\
&+2(y^3-y^2(3+4\rho_\tau)+(5+11\rho_\tau)y+6\rho_\tau(\rho_\tau-3))x_0\\
&+(-2y^3+2y^2(4+3\rho_\tau)-y(15+22\rho_\tau-5\rho_\tau^2)-24\rho_\tau(\rho_\tau-2))x_0^2\\
&+\frac{2}{3}(y^3-y^2(5+2\rho_\tau)+y(10+11\rho_\tau-5\rho_\tau^2)-6\rho_\tau(5-3\rho_\tau))x_0^3\Big)\\
b_0 &= \frac{1}{2\pi}\sqrt{y^2-4\rho_\tau}\left(\frac{3}{2}(y^2-4\rho_\tau)x_0^2-\frac{1}{2}(y^2+y(1+\rho_\tau)-8\rho_\tau)x_0^3\right) && (32)\\
b_1 &= \frac{1}{2\pi}\frac{\sqrt{y^2-4\rho_\tau}}{(1+\rho_\tau-y)^2}\Big((y^2-4\rho_\tau)^2+2(y^2-4\rho_\tau)(y^2-4y(1+\rho_\tau)+12\rho_\tau)x_0 && (33)\\
&-(2y^4-6y^3(1+\rho_\tau)+y^2(-5+8\rho_\tau-5\rho_\tau^2)+44\rho_\tau y(1+\rho_\tau)\\
&+\rho_\tau(10-124\rho_\tau+10\rho_\tau^2))x_0^2\\
&+\frac{2}{3}(y^4-2y^3(1+\rho_\tau)-5y^2(1+\rho_\tau^2)+28\rho_\tau y(1+\rho_\tau)+2\rho_\tau(5-38\rho_\tau+5\rho_\tau^2))x_0^3\Big)
\end{aligned}
$$

In all of these expressions $x_0=1-\frac{\rho_c}{1+\rho_\tau-y}$, $\rho_c=\frac{m_c^2}{m_b^2}$, $\rho_\tau=\frac{m_\tau^2}{m_b^2}$.
We have checked that our result for $d\Gamma$ reproduces the result in [3] for vanishing lepton mass. The part proportional to K in the polarized decay width reproduces the result for B decays in [7].

IV Transverse Polarization in Λ_b decays

As already indicated one encounters certain technical difficulties in the calculation of transverse polarizations if the parametrization in terms of the momentum transfer $\hat{q}^2$ is used. To illustrate the origin of these difficulties we will discuss the transverse τ-polarization for the decays of polarized Λ_b in some detail.

We choose the rest frame of the hadron as our coordinate system and the direction

of the τ emission as z-axis. In this system the vectors take the form

$$P_\tau = \begin{pmatrix} P_\tau^0 \\ 0 \\ 0 \\ |\underline{P}_\tau| \end{pmatrix}, \quad P_\nu = \begin{pmatrix} P_\nu^0 \\ |\underline{P}_\nu| \cos(\phi_\nu)\sin(\Theta_\nu) \\ |\underline{P}_\nu| \sin(\phi_\nu)\sin(\Theta_\nu) \\ |\underline{P}_\nu| \cos(\Theta_\nu) \end{pmatrix}$$

$$s_h = \begin{pmatrix} 0 \\ \cos(\phi_\nu+\psi)\sin(\Theta_s) \\ \sin(\phi_\nu+\psi)\sin(\Theta_s) \\ \cos(\Theta_s) \end{pmatrix}, \quad s_\tau = \begin{pmatrix} 0 \\ \cos(\phi_\nu+\delta) \\ \sin(\phi_\nu+\delta) \\ 0 \end{pmatrix} \tag{34}$$

s_h is the spin vector of the hadron. ψ (δ) is the azimuthal angle between the hadron (lepton) spin and the neutrino momentum. This parametrization of the spin projection vector has the advantage that transverse polarizations in arbitrary orientations relative to the decay plane can be determined. $\delta = 0$ gives the transverse polarization in the decay plane, while $\delta = \frac{\pi}{2}$ corresponds to s_τ perpendicular to that plane. The differential decay width in scaled variables is given by

$$\begin{aligned} \frac{1}{\Gamma_0} d\Gamma^p &= \frac{3}{8m_b\pi} L^p_{\mu\nu} H^{\mu\nu} \delta\left(\hat{q}^2 - \rho_\tau - \frac{1}{2}x(y - \sqrt{y^2-4\rho_\tau}\cos(\Theta_\nu))\right) \\ &\quad \cdot x\sqrt{y^2-4\rho_\tau}\, d\hat{q}^2 dx dy d\psi d\cos(\Theta_s) d\cos(\Theta_\nu) \end{aligned} \tag{35}$$

A δ-function relating $\hat{q}^2$ to $\cos(\Theta_\nu)$ has been included to transform phase space and the matrix element to the same set of variables. The matrix element $L^p_{\mu\nu}T^{\mu\nu}$ may be written symbolically as

$$L^p_{\mu\nu}T^{\mu\nu} = \left(f_1 \frac{1}{\Delta_0} + f_2 \frac{1}{\Delta_0^2} + f_3 \frac{1}{\Delta_0^3}\right) \tag{36}$$

with $f_i = f_i(x, y, \cos(\Theta_\nu), \sin(\Theta_\nu))$. The functions $f_i, i = 1,2,3$ are analytic in the complex x-plane. Each power of Δ_0 contributes a pole of corresponding order. To obtain the decay width (35) as a function of $\hat{q}^2$ instead of $\cos(\Theta_\nu)$ one must integrate over Θ_ν. This integration produces two Heaviside functions

$$\Theta\left(\frac{2(\hat{q}^2-\rho_\tau)}{y-\sqrt{y^2-4\rho_\tau}} - x\right)\Theta\left(x - \frac{2(\hat{q}^2-\rho_\tau)}{y+\sqrt{y^2-4\rho_\tau}}\right) \tag{37}$$

and the replacements

$$\cos(\Theta_\nu) \;\to\; -2\frac{\hat{q}^2-\rho_\tau-\frac{1}{2}xy}{x\sqrt{y^2-4\rho_\tau}} \tag{38}$$

$$\sin(\Theta_\nu) \;\to\; 2\frac{\sqrt{(\hat{q}^2-\rho_\tau)xy - \rho_\tau x^2 - (\hat{q}^2-\rho_\tau)^2}}{x\sqrt{y^2-4\rho_\tau}} \tag{39}$$

In the scaled variables the poles have the form

$$\frac{1}{(1-\rho_c-x-y+\hat{q}^2)^n},\ n=1,2,3 \tag{40}$$

The imaginary part of $L_{\mu\nu}T^{\mu\nu}$ and $L^p_{\mu\nu}T^{\mu\nu}$ can be taken by performing a contour integration around the poles in the complex x-plane. The apparent poles at $x=0$ are compensated by the factor $|P_\nu| = \frac{1}{2}m_b x$ in the neutrino momentum P_ν. Thus only (39) alters the analytic structure of the integrand by introducing two branch cuts in addition to the poles. The analytic structure of the parts containing the square roots is sketched in Fig. 1. D is the position of the pole. The branch cuts are chosen to extend from the points A and B to infinity. C is the contour for the integration. As long as the pole and the ends of the branch cuts are well separated the contour integration can be performed with the residue theorem. This yields the replacement of the poles by δ-functions as a means of taking the imaginary part of the matrix element [3]. A necessary condition for this technique to be applicable is the analyticity of the function multiplying the pole in the vicinity of the pole.

The subsequent integration over $\hat{q}^2$ corresponds to moving the poles and the ends of the branch cuts. For $\hat{q}^2$ equal to its minimal or maximal value in the $\hat{q}^2$ integration, the pole and the end of the left or the right branch cut fall together, i.e $A=D$ or $B=D$. For $\hat{q}^2$ arbitrarily close to these values the contour integral is not defined because the contour is pinched between two non analytic points. Correspondingly the integral over $\hat{q}^2$ is well defined only if the limits of the integral do not approach the maximum or minimum possible values. If the limits are included the integral diverges if one replaces the poles with δ-functions because the necessary condition for that replacement is no longer fulfilled.

For poles of first order only the result stays finite for all values of $\hat{q}^2$ because integrals over first order poles embedded in branch cuts can be defined. The calculation of transverse polarizations in the free quark model produces only poles of first order as can be seen by setting K and ϵ zero in the form factors. Therefore one does not encounter any divergences in that case.

The problem mentioned above can be avoided by integrating (35) over $\hat{q}^2$ instead of $\cos(\Theta_\nu)$. Then the δ-function in (35) yields

$$\Theta(x)\Theta(\sqrt{y^2-4\rho_\tau})\Theta(1-\cos(\Theta_\nu))\Theta(\cos(\Theta_\nu)-1) \tag{41}$$

The replacement (39) is never made, so no branch cuts are introduced. Setting

$$\hat{q}^2 = \rho_\tau + \frac{1}{2}x\left(y-\sqrt{y^2-4\rho_\tau}\cos(\Theta_\nu)\right) \tag{42}$$

in the form factors leads to modified expressions for the poles.

$$\begin{aligned}&\frac{1}{(1-\rho_\tau-x-y+\hat{q}^2)^n}\\ &\quad=\frac{2^n}{(2-y+\sqrt{y^2-4\rho_\tau}\cos(\Theta_\nu))}\frac{1}{\left(2\frac{1-\rho_c+\rho_\tau-y}{2-y+\sqrt{y^2-4\rho_\tau}\cos(\Theta_\nu)}-x\right)^n}\end{aligned} \tag{43}$$

For the maximal value of the τ-energy $y_{max} = 1 + \rho_\tau - \rho_c$ the pole is located at $x = 0$. Strictly speaking a contour integral around the pole is not defined in this case because $\Theta(x)F(x)$, $F(x)$ analytic, does not possess an analytic continuation for $x < 0$. However, since the derivatives of this function are well defined in the sense of distributions, one can treat this expression as analytic at the price of introducing its derivatives, containing δ-functions, into the energy spectra. Substituting

$$\frac{1}{\left(2\frac{1-\rho_c+\rho_\tau-y}{2-y+\sqrt{y^2-4\rho_\tau}\cos(\Theta_\nu)} - x\right)^n} \rightarrow \frac{(-1)^{n-1}}{(n-1)!}\delta^{(n-1)}\left(2\frac{1-\rho_c+\rho_\tau-y}{2-y+\sqrt{y^2-4\rho_\tau}\cos(\Theta_\nu)} - x\right) \qquad (44)$$

under the integral over x to obtain the imaginary part directly leads to a matrix element that can be integrated easily. In this parametrization the expressions multiplying the Heaviside function contain a factor x^2 or a higher power of x together with poles of at most third order. Since for $x \geq 0$ $\Theta(x)x^n$ has the same finite derivatives of up to n-th order as x^n, the Heaviside function can be omitted in all expressions containing poles of up to order $n+1$ [1]. All expressions we consider in this paper fulfill this condition. Thus after integration over x no terms containing δ-functions survive. To illustrate this a decay width differential in y and $\cos(\Theta_\nu)$ is given in the appendix. The integration over x is trivial and the remaining integration can be performed with a standard substitution. It yields a decay rate of the structure

$$\begin{aligned}\frac{1}{\Gamma_0}\frac{d\Gamma}{dy\,d\psi\,d\cos(\Theta_s)} &= A_0 + A_1K + (B_0(1+\epsilon) + B_1K)\cos(\Theta_s) \\ &\quad + (C_0(1+\epsilon) + C_1K)\sin(\Theta_s)\end{aligned} \qquad (45)$$

for unpolarized leptons. The functions A_0, A_1, B_0 and B_1 are the same as in equations (25)(26)(27) and (28).

$$C_0 = \frac{3}{8}\sqrt{1+\rho_\tau-y}\sqrt{y^2-4\rho_\tau}\cos(\psi)\left((2y-4\rho_\tau)x_0^2 + (-y+2\rho_\tau)x_0^3\right) \qquad (46)$$

$$\begin{aligned}C_1 &= \frac{\sqrt{y^2-4\rho_\tau}\cos(\psi)}{(1+\rho_\tau-y)^{3/2}}\left(\frac{1}{2}\left(y^3 - 2\rho_\tau y^2 - 4\rho_\tau y + 8\rho_\tau^2\right)\right. \\ &\quad + \frac{1}{4}(y^3 - 2(5+2\rho_\tau)y^2 + 4\rho_\tau(3\rho_\tau+8)y + 8\rho_\tau(2-7\rho_\tau))x_0 \\ &\quad + \frac{3}{16}(-5y^3 + 2(10+9\rho_\tau)y^2 - 4\rho_\tau(16+7\rho_\tau) - 24\rho_\tau(1-4\rho_\tau))x_0^2 \\ &\quad + \frac{5}{32}(3y^3 - 2(5+6\rho_\tau)y^2 + 4\rho_\tau(9+4\rho_\tau)y \\ &\quad \left. + 8\rho_\tau(1-6\rho_\tau))x_0^3\right)\end{aligned} \qquad (47)$$

[1] If it is not omitted it produces δ-functions with vanishing coefficients. In the parametrization in terms of $\hat{q}^2$ the factors $x^n, n \geq 2$, are missing. Therefore the δ-functions contribute in that case.

For polarized leptons the spin dependent part of the decay width reads

$$
\begin{aligned}
\frac{1}{\Gamma_0}\frac{d\Gamma^p}{dyd\psi d\cos(\Theta_s)} &= a_0 + a_1 K + (b_0(1+\epsilon) + b_1 K)\cos(\Theta_s) \\
&\quad + (c_0(1+\epsilon) + c_1 K)\sin(\Theta_s)
\end{aligned} \tag{48}
$$

$$
a_0 = \sqrt{\rho_\tau}\frac{3}{16}\sqrt{y^2 - 4\,\rho_\tau}\sqrt{1+\rho_\tau - y}\cos(\delta)(y-2)x_0^3 \tag{49}
$$

$$
\begin{aligned}
a_1 &= \sqrt{\rho_\tau}\frac{1}{64}\frac{\sqrt{y^2-4\rho_\tau}\cos(\delta)}{(1+\rho_\tau - y)^{3/2}}\Big(24(y-2)(y^2-4\rho_\tau)x_0 \\
&\quad +4((1-15\rho_\tau)y^2 - 2(1-29\rho_\tau)y + 8((2-11\rho_\tau+2\rho_\tau^2))x_0^2 \\
&\quad +5(-3y^3 + 2(6+5\rho_\tau)y^2 - 4(4+9\rho_\tau)y + 8\rho_\tau(6-\rho_\tau))x_0^3\Big)
\end{aligned} \tag{50}
$$

$$
b_0 = \sqrt{\rho_\tau}\frac{3}{16}(y^2-4\rho_\tau)\sqrt{1+\rho_\tau - y}\cos(\delta)x_0^3 \tag{51}
$$

$$
\begin{aligned}
b_1 &= \sqrt{\rho_\tau}\frac{(y^2-4\rho_\tau)\cos(\delta)}{64(1+\rho_\tau-y)^{3/2}}\Big(24(y^2-4\rho_\tau)x_0 \\
&\quad -12((1+5\rho_\tau)y + 4(1-4\rho_\tau))x_0^2 \\
&\quad -5(3y^2 - 2(3+5\rho_\tau)y - 4(1-6\rho_\tau))x_0^3\Big)
\end{aligned} \tag{52}
$$

$$
c_0 = -\frac{1}{\pi}\sqrt{\rho_\tau}\sqrt{y^2-4\rho_\tau}(1+\rho_\tau-y)\cos(\delta)\cos(\psi)x_0^3 \tag{53}
$$

$$
\begin{aligned}
c_1 &= \sqrt{\rho_\tau}\frac{\sqrt{y^2-4\rho_\tau}}{\pi(1+\rho_\tau-y)}\Big(-2\cos(\delta)\cos(\psi)(y^2-4\rho_\tau)x_0 \\
&\quad +((\cos(\delta)\cos(\psi) - \cos(\delta-\psi))y^2 \\
&\quad +(-(1-3\rho_\tau)\cos(\delta)\cos(\psi) + (3+\rho_\tau)\cos(\delta-\psi))y \\
&\quad +4(1-3\rho_\tau)\cos(\delta)\cos(\psi) - 2(1+\rho_\tau)\cos(\delta-\psi))x_0^2 \\
&\quad +\frac{2}{3}\cos(\delta)\cos(\psi)(2y^2 - 5(1+\rho_\tau)y + 12\rho_\tau)x_0^3\Big)
\end{aligned} \tag{54}
$$

In [8] the same technical problem has been encountered in a somewhat different context. We have checked that our method yields the same result as the regularization procedure introduced in that paper.

V Transverse Polarization in B-meson decays

The calculation of the transverse τ-polarization in the decays of B-mesons proceeds along precisely the same lines. The matrix element is somewhat simplified because of the absence of a hadron spin but the terms containing the chromomagnetic operator can no longer be ignored. Only the parts of $T^{\mu\nu}$ that do not contain the hadron spin contribute to this decay. The necessary equations, definitions and techniques have

been discussed in [3] [4] [7] or in this paper so we will simply present our results for the polarized and unpolarized decay width.

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = 4\pi\left(A_0 + A_1 K + A_2 G\right) \tag{55}$$

The functions A_0 and A_1 are identical to those in the Λ_b decay rate into unpolarized leptons (25)(26), while A_2 is given by

$$\begin{aligned} A_2 \;=\; & \frac{1}{\pi}\frac{\sqrt{y^2-4\rho_\tau}}{1+\rho_\tau-y}\Big(\Big(5y^3-10(1+\rho_\tau)y^2+4(3+4\rho_\tau)y-16\rho_\tau(2-\rho_\tau)\Big)\,x_0 \\ & -\Big(5y^3-(17+15\rho_\tau)y^2+(24+46\rho_\tau)y-70\rho_\tau+18\rho_\tau^2\Big)\,x_0^2 \\ & +\frac{5}{3}\Big(y^3-4(1+\rho_\tau)y^2+2(3+7\rho_\tau)y-4\rho_\tau(5-\rho_\tau)\Big)\,x_0^3\Big) \end{aligned} \tag{56}$$

The decay rate into transversely polarized leptons has the same decomposition

$$\frac{1}{\Gamma_0}\frac{d\Gamma^p}{dy} = 4\pi(a_0 + a_1 K + a_2 G) \tag{57}$$

$$\begin{aligned} a_2 \;=\; & \frac{1}{2}\sqrt{\rho_\tau}\frac{\sqrt{y^2-4\rho_\tau}\cos(\delta)}{\sqrt{1+\rho_\tau-y}}\left(-\left(y-2\right)x_0\right. \\ & +\frac{1}{8}\Big(-15y^2+50y-8(7-2\rho_\tau)\Big)\,x_0^2 \\ & \left.+\frac{5}{16}\Big(5y^2-16y+4(4-\rho_\tau)\Big)\,x_0^3\right) \end{aligned} \tag{58}$$

the functions a_0, a_1 being identical to (49) and (50). The unpolarized decay width (55) reproduces the result of [5] [6]. For the choice $\delta = \frac{\pi}{2}$, i.e. s_τ perpendicular to the decay plane, the transverse polarization vanishes.

VI Numerical Evaluation

For the numerical evaluation of the spectra we have used the following parameters: $m_b = 5.3\,\mathrm{GeV}, m_c = 1.85\,\mathrm{GeV}, m_\tau = 1.777\,\mathrm{GeV}$. The HQET parameters are set to $\epsilon = 0, K = 0.01$ and for meson decays $G = -0.0065$. Unlike the energy spectra the polarizations are finite over the whole kinematically allowed range of the τ-energy. In Fig. 2 the longitudinal τ-polarization in polarized Λ_b decays is shown for the hadron spin perpendicular to the τ-momentum. Other orientations of the hadron spin yield qualitatively similar graphs. For comparison the free quark model (FQM) prediction is included. The HQET corrections are small. Only near the end of the spectrum they become noticeable but in that region the OPE begins to break down. This produces the endpoint divergences in the τ-energy spectra which have

to be removed by applying smoothing functions before the results are physically meaningful. We expect the peak in Fig. 2 to be due to the fact that we used the HQET spectra without smoothing them. If smoothed spectra are used it probably will be less pronounced. For the average polarization $\mathcal{P} = 2\frac{\Gamma^p}{\Gamma}$ we obtain

$$\mathcal{P} = -0.72 \tag{59}$$

This value is in the same range as the longitudinal polarization in B-meson decays calculated in [7].

The transverse polarization in Λ_b decays in shown for two different configurations. Fig. 3 shows it for the hadron spin parallel to the τ spin projection vector and both lying in the decay plane. The HQET corrections to the FQM prediction are small. The dip at high τ-energies is again due to the use of unsmoothed energy spectra. Using smoothed spectra will smear it out. For low τ-energies this polarization is not a small quantity (30%). As expected, it is of the order $\sqrt{\rho_\tau}$. The average polarization for this orientation of the spins is

$$\mathcal{P} = -0.19 \tag{60}$$

If the hadron spin and the spin projection vector are parallel to each other and perpendicular to the decay plane, there are no FQM contributions to the transverse polarization. All of the polarization shown in Fig. 4 is due to the HQET corrections. Furthermore, for this choice of the angles, the expressions containing ϵ vanish, so that the polarization is proportional to $K\sqrt{\rho_\tau}$. For low τ-energies a polarization of approximately 1.5% is expected. The average polarization is

$$\mathcal{P} = -0.011 \tag{61}$$

It may be recalled that a small non-zero polarization transverse to the decay plane can also be induced by final state interactions. That effect, however, is independent of the spin-orientation of the decaying particle.

The transverse polarization in B-meson decay is strongest if the spin projection vector is chosen in the decay plane. Fig. 5 shows this polarization. In this case the corrections proportional to G have to be included but parts involving the hadron spin are absent. The prediction is dominated by the FQM contribution of the order $\sqrt{\rho_\tau}$. The dip at high energies is mostly due to the fact that we use unsmoothed HQET spectra. The average value for this transverse polarization is

$$\mathcal{P} = -0.2 \tag{62}$$

Again, for low τ-energies, the polarization is rather large (40%).

Appendix

As an example of the fact that no δ-functions appear in the two dimensional decay widths if one parametrizes in terms of $\cos(\Theta_\nu)$ we will consider the rate for $B \to$

$X_c e^- \bar{\nu}$ for unpolarized electrons. It can be decomposed as

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy\, d\cos(\Theta_\nu)} = D_0 + D_1 K + D_2 G \tag{63}$$

The functions D_0, D_1 and D_2 are given by

$$\begin{aligned}
D_0 &= 48y^2(1-\rho_c-y)^2\frac{1+\rho_c+\cos(\Theta_\nu)(1-\rho_c)}{(2-y+y\cos(\Theta_\nu))^4} && (64)\\
D_1 &= \frac{32y^2}{(2-y+y\cos(\Theta_\nu))^6}\Big(-2y^3(1-\rho_c)(1-\rho_c-y)\cos^4(\Theta_\nu) && (65)\\
&\quad +y^2((10+2\rho_c)y^2+(18\rho_c+3\rho_c^2-21)y+12(1-rj)^2)\cos^3(\Theta_\nu)\\
&\quad y((-10+6\rho_c)y^3+(21-24\rho_c+3\rho_c^2)y^2+12(\rho_c^2-1)y^2\\
&\quad\ +4(1+3\rho_c-9\rho_c^2+5\rho_c^3))\cos^2(\Theta_\nu)\\
&\quad (-(18+10\rho_c)y^4+(53-22\rho_c-7\rho_c^2)y^3-12(5-6\rho_c+5\rho_c^2)y^2\\
&\quad\ +4(11-16\rho_c+15\rho_c^2-10\rho_c^3)y-16(1-\rho_c)^2)\cos(\Theta_\nu)\\
&\quad +y(4(4+\rho_c)y^3-(51-24\rho_c-3\rho_c^2)y^2+12(5-4\rho_c+3\rho_c^2)y\\
&\quad\ -4(6-7\rho_c+6\rho_c^2-5\rho_c^3))\Big)\\
D_2 &= \frac{32y^2(1-\rho_c-y)}{(2-y+y\cos(\Theta_\nu))^5}\Big(2y^2(\rho_c-2)\cos^3(\Theta_\nu) && (66)\\
&\quad +y(2-7y-\rho_c y-10\rho_c)\cos^2(\Theta_\nu)\\
&\quad +((6-4\rho_c)y^2-4y+12\rho_c-20\rho_c^2)\cos(\Theta_\nu)\\
&\quad +((5+3\rho_c)y^2+(2+10\rho_c)y-4+8\rho_c+20\rho_c^2)\Big)
\end{aligned}$$

This decay width is free of δ-functions. It can be compared directly with the corresponding spectrum differential in y and $\hat{q}^2$ (eqn. 5.2 in [3]) which contains δ-functions and their first derivatives. After integration over $\cos(\Theta_\nu)$ and $\hat{q}^2$ respectively the results for the lepton energy spectrum $d\Gamma/dy$ coincide.

References

[1] J. Chay, H. Georgi, B.Grinstein, Phys. Lett. B247, 299 (1990)

[2] I.Bigi, M. Shifman, N. Uraltsev, A. Vainshtein, Phys. Rev. Lett. 71, 496 (1993)

[3] A. V. Manohar and M.B. Wise, Phys. Rev. D49, 1310 (1994)

[4] B. Blok, L. Koyrakh, M. Shifman and A.I. Vainshtein, Phys. Rev. D49, 3356 (1994)

[5] L. Koyrakh, Phys. Rev. D49, 3379 (1994)

[6] S. Balk, J. G. Körner, D. Pirjol, K. Schilcher, Z. Phys. C64, 37 (1994)

[7] A.F. Falk, Z. Ligeti, M. Neubert, Y. Nir, Phys. Lett. B326, 145 (1994)

[8] Y. Grossman, Z. Ligeti, hep-ph/9409418

[9] I. Bigi, M. Shifman, N.G. Uraltsev, A. Vainshtein, CERN-TH.7250/94 and references therein

Figure Captions

Fig. 1 Branch cuts in the complex x-plane

Fig. 2 Longitudinal Polarization: $\Theta_s = \frac{\pi}{2}$, Λ_b decay

Fig. 3 Transverse Polarization: $\Theta_s = \frac{\pi}{2}, \delta = 0, \psi = 0$, Λ_b decay

Fig. 4 Transverse Polarization: $\Theta_s = \frac{\pi}{2}, \delta = \frac{\pi}{2}, \psi = \frac{\pi}{2}$, Λ_b decay

Fig. 5 Transverse Polarization: $\delta = 0$, B decay

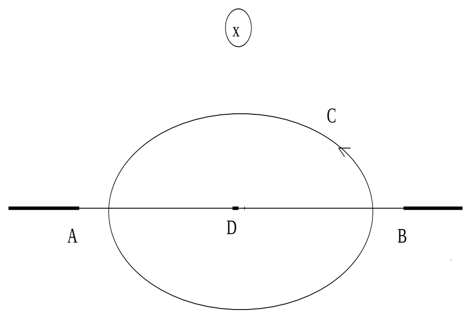


Figure 1: .

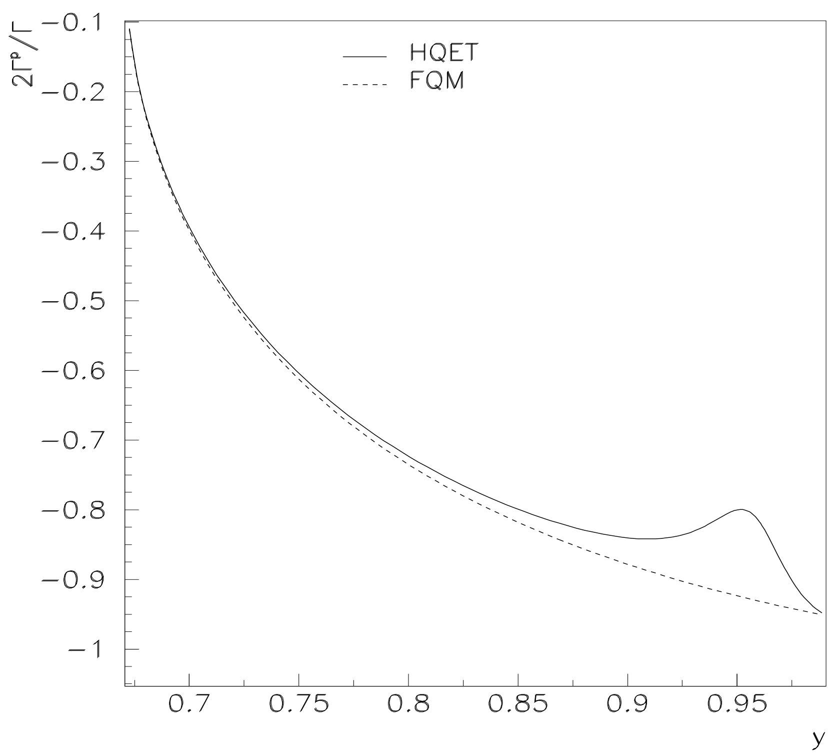
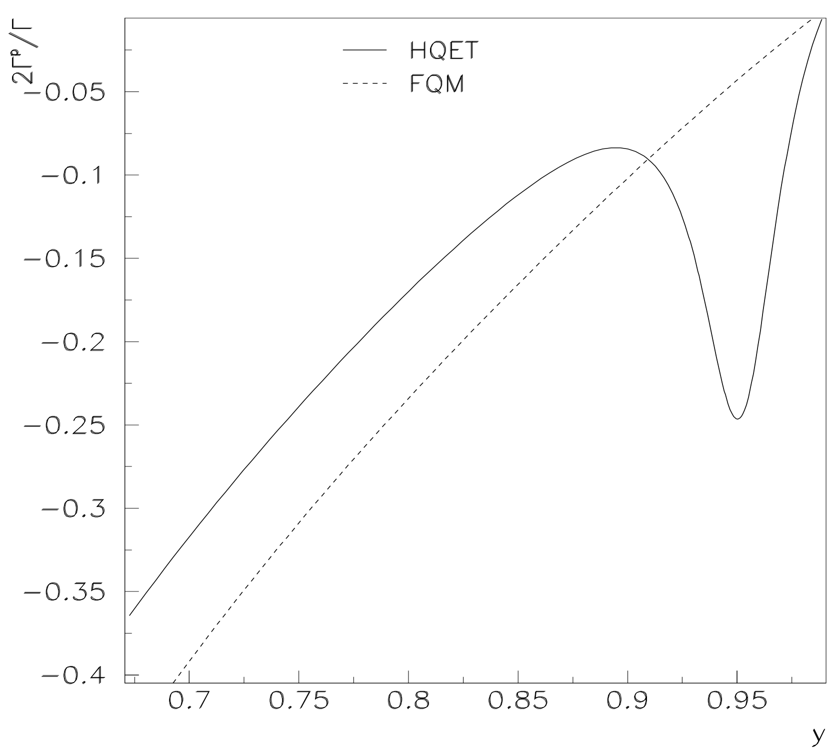


Figure 2: .



HQET
FQM
$2\Gamma^p/\Gamma$
−0.05
−0.1
−0.15
−0.2
−0.25
−0.3
−0.35
−0.4
0.7
0.75
0.8
0.85
0.9
0.95
y

Figure 3: .

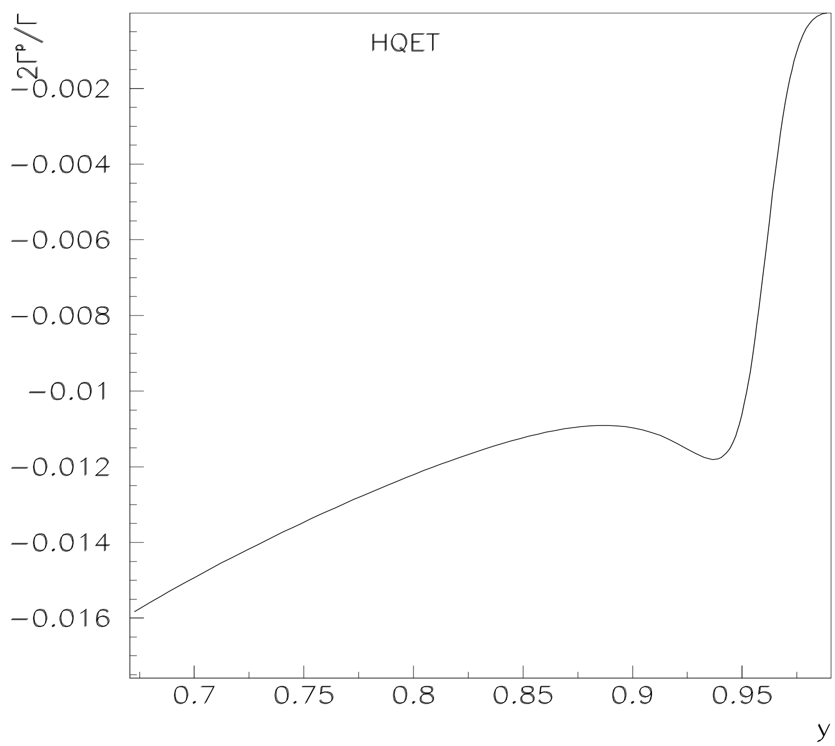

HQET
−0.002
−0.004
−0.006
−0.008
−0.01
−0.012
−0.014
−0.016
0.7
0.75
0.8
0.85
0.9
0.95
y

Figure 4: .

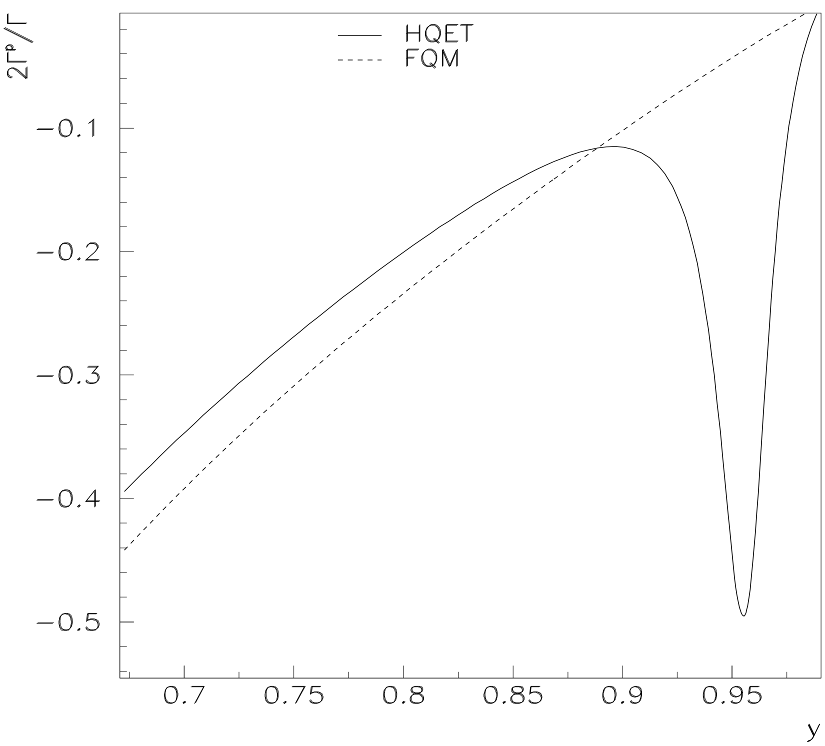


Figure 5: .